position=15,-0.25, angle=0, vshift=0, hshift=0, opacity=1, scale=1.1,

contents=

1	2	3	4	5	6
(15,1)					

, color=black, firstpage=true,

Sign each of the pages!

Name and Last name: ________________________________

1. Explain why a given system $\frac{\partial U}{\partial t} + \mathbb{A}\frac{\partial U}{\partial x} = 0$ is hyperbolic or not? Provide the explanation on this page, next to the problem!

   (a) (3 points) $\mathbb{A} = \begin{bmatrix} 5 & 1 \\ 6 & 4 \end{bmatrix}$

   (b) (3 points) $\mathbb{A} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

   (c) (3 points) $\mathbb{A} = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

   (d) (3 points) $\mathbb{A} = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

2. (12 points) Solve hyperbolic system: $\begin{cases} \frac{\partial U}{\partial t} + \mathbb{A}\frac{\partial U}{\partial x} = 0 \\ U(x, t = 0) = \begin{bmatrix} sin(2x) \\ 0 \end{bmatrix} \\ t > 0 \end{cases}$, $\mathbb{A} = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$

3. (12 points) What is the time and position at which the discontinuity appears? What is the speed with which it travels afterwards? Prepare a drawing (15cm $\times$ 15cm) illustrating characteristic curves on the time space plane. $\begin{cases} \frac{\partial U}{\partial t} + \frac{\partial}{\partial x}(\frac{1}{4}u^2) = 0 \\ U(x, t = 0) = \begin{cases} 1 & |x| > 1 \\ -x & x \in < -1, 0 > \\ x & x \in < 0, 1 > \end{cases} \end{cases}$

4. (12 points) For the equation of question 3, plot solution at $t = \frac{3}{4}$. Calculate the speed with which the discontinuity travels.
   Assume initial condition to be: $U(x, t = 0) = \begin{cases} -3 & |x| > 2 \\ 0 & x \in < -2, 2 > \end{cases}$

5. (12 points) Propose an iterative solution strategy, based on quasi linearisation for a given nonlinear boundary problem:
   $\begin{cases} \triangle u - u^2(u - 2) = 0 \\ u|_\Omega = 1 \qquad \Omega = < 0, 1 > \times < 0, 1 > \end{cases}$