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Name and Last name: _____

1. Explain why a given system $\frac{\partial U}{\partial t} + \mathbb{A} \frac{\partial U}{\partial x} = 0$ is hyperbolic or not? Provide the explanation on this page, next to the problem!

(a) (2½ points) $\mathbb{A} = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

(b) (2½ points) $\mathbb{A} = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}$

(c) (2½ points) $\mathbb{A} = \begin{bmatrix} 6 & 1 \\ 4 & 6 \end{bmatrix}$

(d) (2½ points) $\mathbb{A} = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

2. (10 points) Solve hyperbolic system:
$$\begin{cases} \frac{\partial U}{\partial t} + \mathbb{A} \frac{\partial U}{\partial x} = 0 \\ U(x, t = 0) = \begin{bmatrix} 0 \\ \cos(2x) \end{bmatrix}, \mathbb{A} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \\ t > 0 \end{cases}$$

3. (10 points) What is the time and position at which the discontinuity appears? What is the speed with which it travels afterwards? Prepare a drawing (15cm × 15cm) illustrating characteristic curves on the time space plane.

$$\begin{cases} \frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left(-\frac{3}{4} u^2 \right) = 0 \\ U(x, t = 0) = \begin{cases} 1 & |x| > 1 \\ -x & x \in \langle -1, 0 \rangle \\ x & x \in \langle 0, 1 \rangle \end{cases} \end{cases}$$

4. (10 points) For the equation of question 3, plot solution at $t = \frac{5}{4}$. Calculate the speed with which the discontinuity travels.

Assume initial condition to be:
$$U(x, t = 0) = \begin{cases} -3 & |x| > 2 \\ 0 & x \in \langle -2, 2 \rangle \end{cases}$$

5. (10 points) Propose an iterative solution strategy, based on quasi linearisation for a given nonlinear boundary problem:

$$\begin{cases} \Delta u - e^{-u} = 0 \\ u|_{\Omega} = 1 \end{cases} \quad \Omega = \langle 0, 1 \rangle \times \langle 0, 1 \rangle$$