Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and Last name: ____________________

1. Explain why a given system $\frac{\partial U}{\partial t} + \mathbb{A}\frac{\partial U}{\partial x} = 0$ is hyperbolic or not?
   (a) (2½ points) $\mathbb{A} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$
   (b) (2½ points) $\mathbb{A} = \begin{bmatrix} 5 & 0 \\ 1 & 5 \end{bmatrix}$
   (c) (2½ points) $\mathbb{A} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
   (d) (2½ points) $\mathbb{A} = \begin{bmatrix} 2 & 1 \\ 6 & 4 \end{bmatrix}$

2. (10 points) Solve a hyperbolic system for $t > 0$ $\begin{cases} \frac{\partial U}{\partial t} + \mathbb{A}\frac{\partial U}{\partial x} = 0 \\ U(x, t = 0) = \begin{bmatrix} 0 \\ cos(x) \end{bmatrix} \end{cases}$, $\mathbb{A} = \begin{bmatrix} 2 & 2 \\ -2 & 3 \end{bmatrix}$

3. (10 points) How long a solution to a nonlinear advection problem yields a classical solution (there are no discontinuities). $\begin{cases} \frac{\partial U}{\partial t} + \frac{\partial}{\partial x}(-4u^2) = 0 \\ U(x, t = 0) = \begin{cases} 1 & |x| > 1 \\ -x & x \in < -1, 0 > \\ x & x \in < 0, 1 > \end{cases} \end{cases}$, $\boxed{\text{t=}\qquad}$

4. (10 points) For the equation of question 3, draw a solution for $t = \frac{1}{4}$, assume initial condition: $U(x, t = 0) = \begin{cases} 1 & |x| > 1 \\ -x & x \in < -1, 0 > \\ x & x \in < 0, 1 > \end{cases}$

5. (10 points) Propose an iterative solution strategy, based on quasi linearisation for a given nonlinear boundary problem:
   $\begin{cases} \triangle u - e^{-u} = 0 \\ u|_\Omega = 1 \qquad \Omega = < 0, 1 > \times < 0, 1 > \end{cases}$